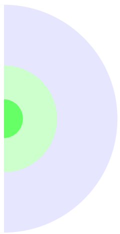
Decibel: $G_{\mathrm{dB}} = 10\log_{10}\left(\frac{P}{P_0}\right)\mathrm{dB}$

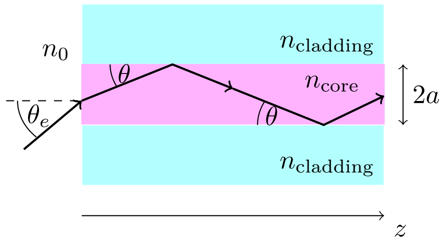
coating
cladding
core



Typical step-index fiber

Numerical Aperture $\mathrm{NA} = n_0 \sin(\theta_{e,\,\max})$
$= \sqrt{n_{\mathrm{core}}^2 - n_{\mathrm{cladding}}^2}$

Guidance via total internal reflection
$\Rightarrow \cos(\theta_{\max}) = \frac{n_{\mathrm{cladding}}}{n_{\mathrm{core}}}$



Phase-relations of the guided rays have to be considered:

$$\Rightarrow \left(kn_{\mathrm{core}}\frac{2a}{\sin(\theta)} + 2\Phi\right) - kn_{\mathrm{core}}2a\left(\frac{1}{\sin(\theta)} - 2\sin(\theta)\right) = m\,2\pi \quad , m \in \mathbb{Z} \quad .$$

where $\Phi_{\mathrm{TE}} = -2\arctan\left(\frac{\sqrt{n_{\mathrm{core}}^2\cos^2(\theta) - n_{\mathrm{cladding}}^2}}{n_{\mathrm{core}}\sin(\theta)}\right)$, $\Phi_{\mathrm{TM}} = -2\arctan\left(\frac{n_{\mathrm{core}}\sqrt{n_{\mathrm{core}}^2\cos^2(\theta) - n_{\mathrm{cladding}}^2}}{n_{\mathrm{cladding}}^2\sin(\theta)}\right)$. $k = \frac{2\pi}{\lambda}$

Normalized frequency / V-number : $V = k_0 a\sqrt{n_{\mathrm{core}}^2 - n_{\mathrm{cladding}}^2} = k_0 a\,\mathrm{NA}$

Radially symmetric step index fibers:

Helmholtz equation: $\Delta u + k_0^2 n^2 u = 0$ with 6 components $[\vec{E},\vec{H}]$; typically E_z, H_z [all other derivable from these two via Maxwell]. $E_z = 0 \Rightarrow$ TE-Modes, $H_z = 0 \Rightarrow$ TM-Modes, $E_z \neq 0$ and $H_z \neq 0 \Rightarrow$ EH- / HE- hybrid modes.
[For $\frac{n_1}{n_2} \approx 1 \Rightarrow$ simplified description: LP-modes]

Guided modes are harmonic [oscillatory] in the core region and exponentially damped [evaneszent] in the cladding.

Mono mode condition $V < V_c = 2{,}405$, total number of guided modes $M \approx 4\left[\frac{V}{\pi}\right]^2 + 2\ [\approx \frac{1}{2}V^2]$ for $V \gg 1$.

[Normalized] propagation constant $b = \frac{\frac{\beta}{k_0} - n_2}{n_1 - n_2}$, with the effective refractive index $n_{\mathrm{eff}} = \frac{\beta}{k_0}$ for the respective mode to which β corresponds.

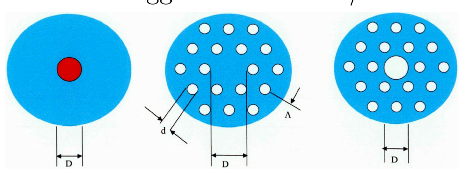
Birefringence:

It means different properties for different polarization directions.

Birefringence $B = \frac{\beta_x - \beta_y}{k_0}$, extinction contrast $E = -10\log_{10}\left(\frac{P_x}{P_x + P_y}\right)$.

[geometrically induced: $B_g = n_1\Delta^2\varepsilon G(V)$ with ellipticity $\varepsilon = \frac{a_x - a_y}{a_x}$ [a_i - core dimensions] and $G(V)$ fiber-dependent; stress-induced: $B_s = [C_2 - C_1][\sigma_x - \sigma_y]$ with $n_i = n_{i0} - C_1\sigma_i - C_2[\sigma_j + \sigma_k]$, $i \neq j \neq k$.]

For very high power purposes e.g. it exist hollow core fibers [guidance via dielectric Bragg reflections and/or metallic mirrors; "leaky waveguides"].



conventional f. holey f. hollow f.

holey f. = "microstructured fiber"

hollow f. = "photonic bandgap fiber"

Show special properties [bandgaps, dispersion, number of modes, ...].

Core can't be made arbitrarily small [→ losses in cladding too high].

Attenuation in a fiber originates from absorption and scattering.

Sellmaier formula: $$n(\lambda) = \sqrt{1 + \sum_{i=1}^{3} A_i \frac{\lambda^2}{\lambda^2 - B_i^2}}$$

$$[\Rightarrow \frac{\mathrm{d}n}{\mathrm{d}\lambda} = -\frac{1}{n}\sum_{i=1}^{3}\frac{A_i B_i^2\lambda}{[\lambda^2 - B_i^2]^2} \quad , \quad \frac{\mathrm{d}^2 n}{\mathrm{d}\lambda^2} = -\frac{1}{n^3}\left[\sum_{i=1}^{3}\frac{A_i B_i^2\lambda}{[\lambda^2 - B_i^2]^2}\right]^2 + \frac{1}{n}\sum_{i=1}^{3} A_i B_i^2 \frac{3\lambda^2 + B_i^2}{[\lambda^2 - B_i^2]^3}]$$

Eikonal equation: $\frac{\mathrm{d}}{\mathrm{d}s}\left[n\frac{\mathrm{d}\vec{r}}{\mathrm{d}s}\right] = \vec{\nabla}n$, refractive index n, ray path s, position $\vec{r}$.

With τ the group delay between in- and output of a fiber [$\tau = \frac{L}{v_{\text{group}}}$] and length L, the **dispersion** is $D = \frac{1}{L}\frac{\mathrm{d}\tau}{\mathrm{d}\lambda_0}$.

For the mode dispersion it is [first order approximation in λ_0] $D_{\text{mode}} = -\frac{\lambda_0}{c_0}\frac{\mathrm{d}^2 n}{\mathrm{d}{\lambda_0}^2} = -\frac{\omega}{\lambda_0}\underbrace{\frac{\mathrm{d}^2\beta}{\mathrm{d}\omega^2}}_{=\beta_2}$, $\beta = \frac{2\pi}{\lambda_0}n_{\text{eff}}$.

With $\beta_m = \left.\frac{\mathrm{d}^m\beta}{\mathrm{d}\omega^m}\right|_{\omega=\omega_0}$ it is $\beta_1 = \frac{1}{v_{\text{group}}} = \frac{n_{\text{eff}}}{c_0}$, $n_{\text{group}} = n - \lambda_0\frac{\mathrm{d}n}{\mathrm{d}\lambda_0}$.

Nonlinear refractive index: Kerr effect $n = n_0 + n_2 I$, intensity I. Figure of merit $[IL] = \int_{-\infty}^{+\infty} I(z)\,\mathrm{d}z$.

The according phase shift is $\Delta\Phi = k_0 n_{\text{eff}} L = \underbrace{k_0 n_0 L}_{\Phi_{\text{linear}}} + \underbrace{\overbrace{\frac{k_0 n_2}{A_{\text{eff}}}}^{=\gamma} PL}_{\Phi_{\text{non-linear}}}$ with power P and effective crosssection area A_{eff}.

With a slowly varying amplitude approximation $u = A\,\mathrm{e}^{\mathrm{i}[\beta z - \omega t]}$ it follows from the scalar, paraxial wave equation $\Delta u + \frac{n^2}{c_0^2}\frac{\partial^2 u}{\partial t^2} = 0$ the so called "nonlinear Schrödinger equation" $i\frac{\partial A}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \gamma\,|A|^2 A = 0$.
[$\beta_1 \approx 0,\ \beta_3 = 0,\ \beta_4 = 0, \ldots$]

Nonlinear $\chi^{(3)}$-effects can lead to self phase modulation, self focussing, cross phase modulation [cross talk of modes], higher frequency generation [multiple photon processes].

Nonlinear effects can cancel the dispersion out → stable light fields [solitons].

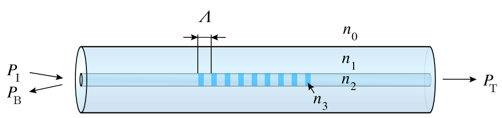
Special fiber modulus: ray transit time [ray picture] $t = \frac{1}{c_0}\int_0^l \frac{n(r,z)}{\cos(\theta_z(r,z))}\,\mathrm{d}z$.

Beat length of two modes with refractive indices n_{eff1} and n_{eff2} [w.o.l.g. $n_{\text{eff1}} > n_{\text{eff2}}$] is $z_B = \frac{\lambda}{n_{\text{eff1}} - n_{\text{eff2}}} = \frac{2\pi}{\beta_1 - \beta_2}$.

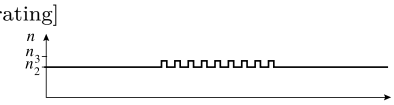
Fiber-coupling:

- by removing the cladding and coating around the core and by bringing two fiber cores very near to one another, a coupling of energy is possible
- in the simplest case a description via a weighted superposition of guided modes of the two coupled waveguides
- the resulting differential equations are very similar to Rabi-oscillations [→ energy is transfered back and forth; depending on the coupling [distance and similarity of the fibers] partial to full transfer of energy]

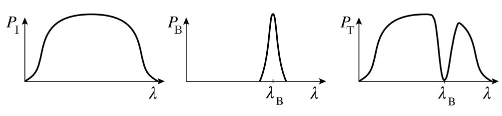
Fiber Bragg grating [FBG]:



- reflected wavelength $\lambda_{B,m} = 2n_{\text{eff}}\Lambda\frac{1}{m}$, $m \in \mathbb{N}$
  [with n_{eff} the effective refractive index of the respective mode in the position of the grating]



- maximal reflected intensity $P_B(\lambda_{B,1}) \approx \tanh^2\left(\frac{N\eta[n_3 - n_2]}{n_{\text{eff}}}\right)$
  [with η the fraction of power in the core, N the number of periods in the grating]
- spectral width of the reflected signal $\Delta\lambda \approx \frac{2[n_3 - n_2]\eta}{\pi}\lambda_B$
  [between first minima besides the maximum at $\lambda_{B,1}$]



Bitrate-length-product: $\text{BL} = \frac{\text{bits}}{\text{second}} \cdot \text{distance}$ [telegraph ~ $10\,\frac{\text{bit}}{\text{s}}$, telephone ~ $10^5\,\frac{\text{bit}}{\text{s}}$, optical fiber ~ $10^{10}\,\frac{\text{bit}}{\text{s}}$]

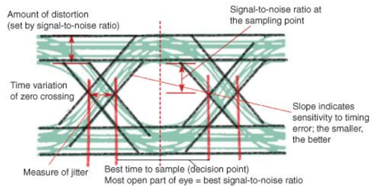
For sampling:

- fullfill Nyquist [$f_{\text{sample}} > 2\Delta f_{\text{signal}}$]
- vertical discretization steps $M = 2^m > \frac{A_{\max}}{A_{\text{noise}}}$

- signal to noise ratio: $\mathrm{SNR} = 10\log\left(\left[\frac{A_{\mathrm{max}}}{A_{\mathrm{noise}}}\right]^2\right)\mathrm{dB}$
- Minimal bitrate for real time transmission:

$$B = m f_{\mathrm{sample}}\mathrm{bit} > \overbrace{\frac{\log_2\left(\frac{A_{\mathrm{max}}}{A_{\mathrm{noise}}}\right)}{10}}^{\approx 0{,}332} \Delta f\, \mathrm{SNR}\, \frac{\mathrm{bit}}{\mathrm{dB}}$$

Eye-diagram: random transmitted signals are recorded and displayed overlayed [e.g. in an oscilloscope]



Bit error rate $\mathrm{BER} = p(1)P(0|1) + p(0)P(1|0)$, [typ. $\sim 10^{-9}$]

where $p(x)$ is the probability for the sending of x and $P(y|x)$ the probability for measuring an y when an x was sent.